Varianta 1

Subiectul I

a) $\frac{x-2}{4}=\frac{y+5}{-6}=\frac{z-3}{9}$. b) 1. c) $\begin{pmatrix} \frac{2\sqrt{10}}{3} & \frac{2\sqrt{10}}{15} \\ \frac{-2\sqrt{10}}{3} & \frac{-2\sqrt{10}}{15} \end{pmatrix}$. d) 0. e) 0. f) 5.

Subiectul II

1. a)1. b) $A_4^3 = 24$. c) 0. d) 6. e) $\frac{1}{2}$.

2 .a) Calcul direct. b) $f'(x)=-\frac{1}{x^2}+\frac{1}{(x+1)^2}$; c) $f'(1)=-\frac{3}{4}$; d) $\ln\frac{4}{3}$. e) 1.

Subiectul III

a) $\det A=-1\neq 0 \Rightarrow rang A=2$; b) $F_2=1, F_3=2$.

c) $A^2=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}=A+I_2$; $A^{n+1}=A^n+A^{n-1}$ se obtine inmultind relatia precedenta cu A^{n-1}.

d) $A^{n+1}=A^n\cdot A=\begin{pmatrix} F_{n+1}+F_n & F_{n+1} \\ F_n+F_{n-1} & F_n \end{pmatrix}=\begin{pmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{pmatrix}$.

e) $(\det A)^n=(-1)^n=\det(A)^n=F_{n+1}\cdot F_{n-1}-F_n^2$.

f) $A^n\cdot A^m=A^{n+m}$ si inlocuim pe A^n, A^m si A^{n+m} din d).

g) $\sum_{k=1}^{n}\frac{(-1)^{k+1}}{F_k\cdot F_{k+1}}\overset{\mathbf{e)}}{=}\sum_{k=1}^{n}\frac{F_k^2-F_{k-1}\cdot F_{k+1}}{F_k\cdot F_{k+1}}=\sum_{k=1}^{n}\left(\frac{F_k}{F_{k+1}}-\frac{F_{k-1}}{F_k}\right)=\frac{F_n}{F_{n+1}}, (\forall) n\geq 1$.

Subiectul IV

a) $g'(x)=-\frac{x}{x+1}$;

b) $x>0, x+1>0, \forall x\in(0,\infty)\overset{\mathbf{a)}}{\Rightarrow} g'(x)<0, \forall x\in(0,\infty)\Rightarrow$ g strict descrescatoare pe $(0,\infty)\Rightarrow g(x)<g(0); \forall x\in(0,\infty)\Rightarrow g(x)<0, \forall x\in(0,\infty)$.

c) $f(x)<1 \Leftrightarrow \frac{\ln(1+x)-x}{x}<0 \Leftrightarrow \frac{g(x)}{x}<0 \Leftrightarrow g(x)<0, \forall x \in (0,\infty)$.

d) Din b) $\Rightarrow x\ln(1+x)<x^2, \forall x \in (0,\infty) \Rightarrow \int_0^1 x\ln(1+x)dx < \int_0^1 x^2 dx \Rightarrow \int_0^1 x\ln x(1+x)dx < \frac{1}{3}$.

e) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1; \lim_{x\to\infty} \frac{\ln(1+x)}{x} = 0.$

f) Fie F : $[0,\infty) \to \mathbf{R}$ o primitiva a functiei f. Avem:

$$\lim_{x\to 0} \frac{\int_{ax}^{bx} f(t)dt}{x} = \lim_{x\to 0} \frac{F(bx)-F(ax)}{x} \underset{l'Hospital}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0} \frac{bf(bx)-af(ax)}{1} =$$

$$= \lim_{x\to 0} \frac{b\ln(1+bx)}{bx} - \frac{a\ln(1+ax)}{ax} = b-a$$

g) f continua pe $[ax,bx]$ implica, $\exists u_x \in (ax,bx)$ astfel incat

$\int_{ax}^{bx} f(t)dt = (bx-ax)\cdot f(u) = x(b-a)f(u_x)$, $x \to \infty$ implica $u_x \to \infty$. Avem:

$$\lim_{x\to\infty} \frac{1}{x}\int_{ax}^{bx} f(t)dt = \lim_{u_x\to\infty} (b-a)f(u_x) = (b-a)\lim_{u_x\to\infty} f(u_x) = (b-a)\cdot 0 = 0.$$